Will make effort to stay on syllabus numbering.

ECE 440: Lecture 12

Diffusion Current

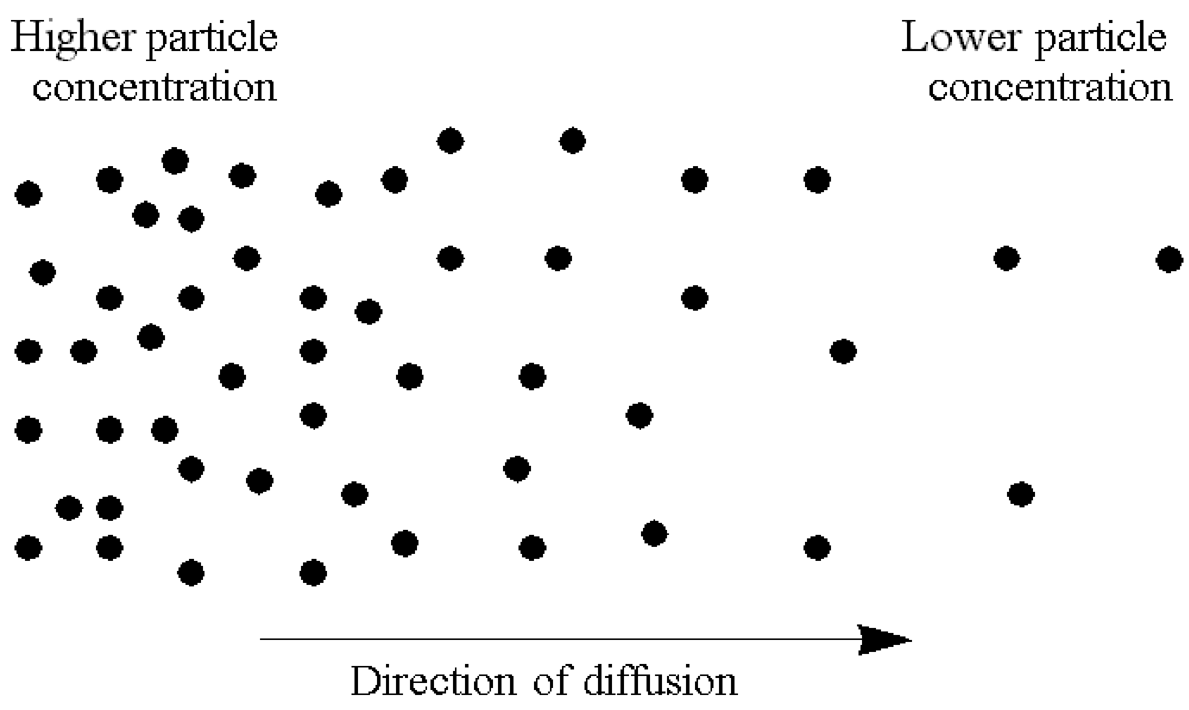
Remember Brownian motion of electrons & holes!

When E field = 0, but finite temperature (T > 0) thermal agitation velocity v_T =

But net drift velocity v_d =

So is net current J_d = =

Now, what if there is a *concentration* gradient or a Brownian *velocity* gradient?



Is there a net flux of particles? Is there net current?

Examples of diffusion:

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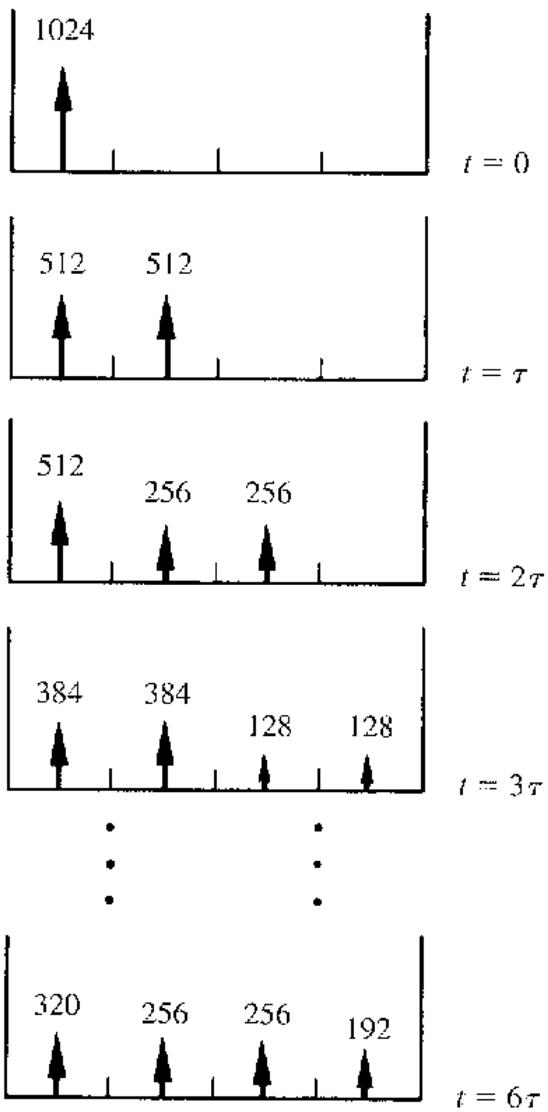
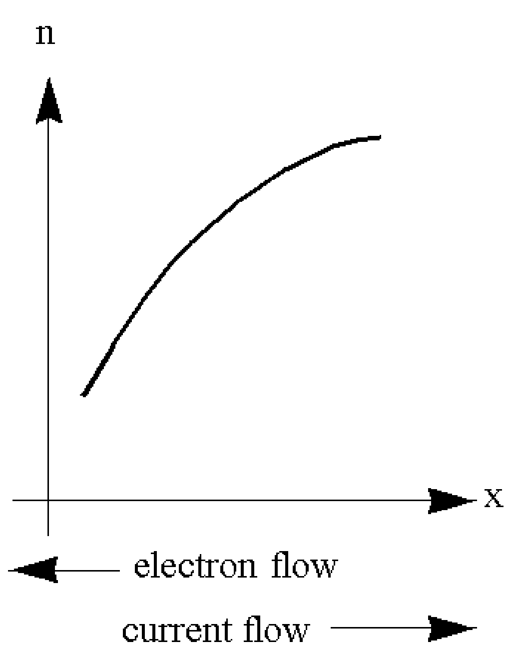
-
-

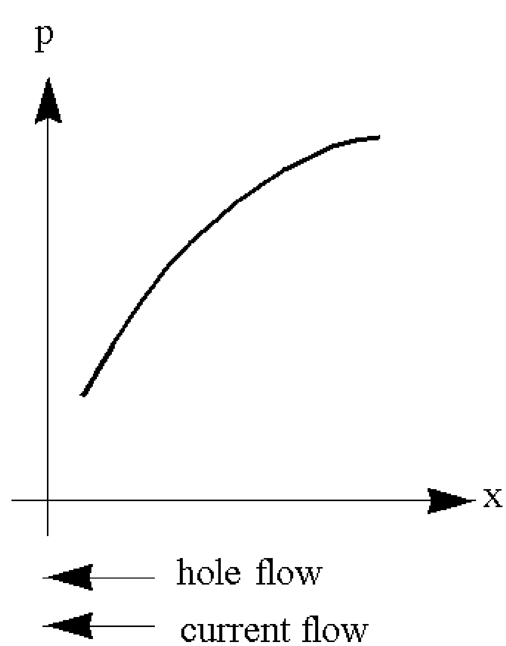
One-dimensional diffusion example:



How would you set up diffusion in a semiconductor? You need something to drive it out of equilibrium.

What drives the *net* diffusion current? The concentration gradients! (no n or p gradients, no net current).





Mathematically:

$J_{N,diff}$ = **$J_{P,diff}$ =**

Where D_N and D_P are the *diffusion coefficients* or *diffusivity*.

Now, we can FINALLY write down the TOTAL current in a semiconductor:

For electrons:

$$J_N = J_{N,drift} + J_{N,diff} = qn\mu_n \mathcal{E} + qD_N \frac{dn}{dx}$$

For holes:

$$J_P = J_{P,drift} + J_{P,diff} = qp\mu_p \mathcal{E} - qD_P \frac{dp}{dx}$$

And TOTAL current:

$$J =$$

Interesting point: minority carriers contribute little to drift current (usually, too few of them!), BUT if their gradient is high enough…

Under equilibrium, or open circuit conditions, total current must always be =

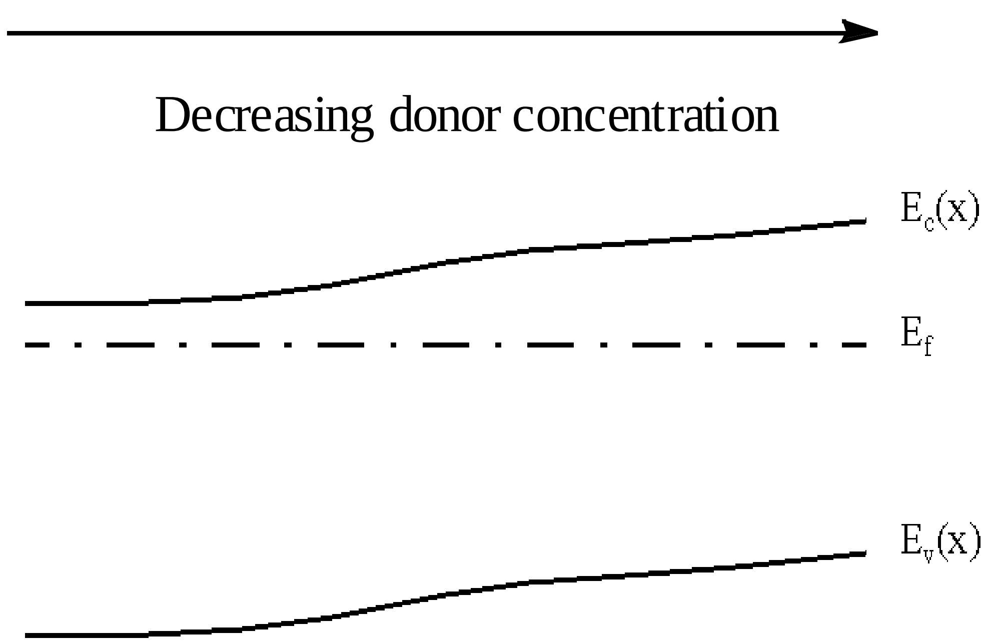
I.e. $J_{drift} = -J_{diffusion}$

More mathematically, for electrons:

So, any disturbance (e.g. light, doping gradient, thermal gradient) which may set up a carrier concentration gradient, will also internally set up a built-in ___________________

What's the relationship between mobility and diffusivity?

Consider this band diagram (see, these things are *useful*):



Going back to *Drift* + *Diffusion* = 0 in equilibrium:

$$J_{\mathrm{N}} = qn\mu_n\mathcal{E} + qD_N\frac{dn}{dx} = 0$$

Leads us to the <u>Einstein Relationship</u>:

$$\frac{D}{\mu} = \frac{kT}{q}$$

This is very, very important because it connects diffusivity with mobility, which we already know how to look up. Plus, it rhymes in many languages so it's easy to remember.

The Einstein Relationship (almost) always holds true.

So we'll assume this in ECE 440.

<u>Example</u>:

The hole density in an n-type silicon wafer ($N_d = 10^{17}$ cm^{-3}) decreases linearly from 10^{14} cm^{-3} to 10^{13} cm^{-3} between $x = 0$ and $x = 1$ mm (why?). Calculate the hole diffusion current.

PS: we've been surreptitiously neglecting something.

<u>ECE 440: Lecture 13/14</u>

Diffusion with Recombination

Last time:

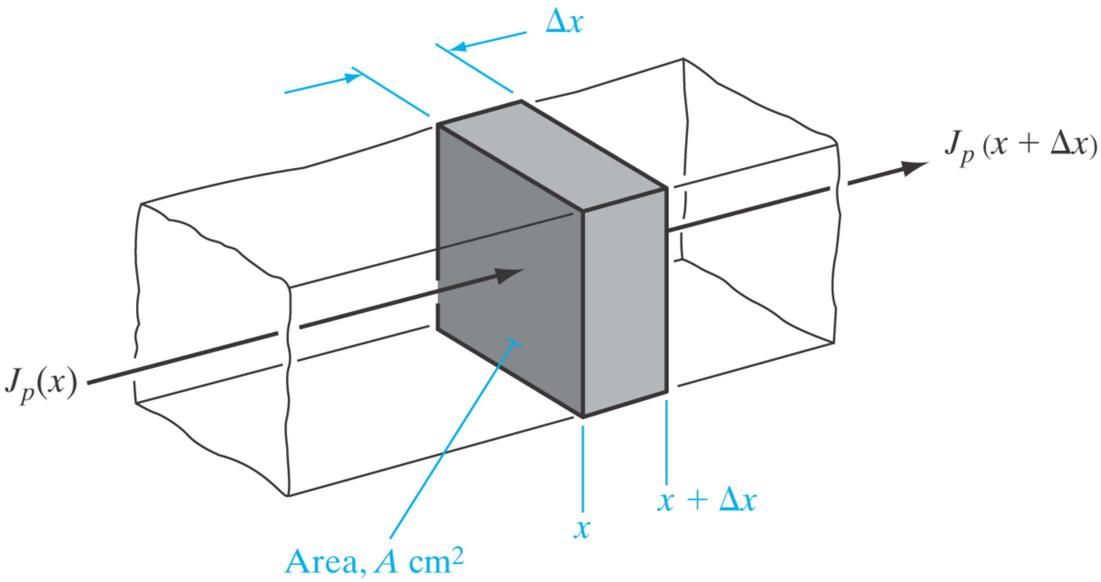
- **Diffusion without recombination (driven by dn/dx)**
- **Einstein relationship (D/μ = kT/q)**
- **kT/q at room temperature ~ 0.026 V (be careful at temperatures different from 300 K)**
- **Mobility μ look up in tables, get diffusivity (be careful with <u>total</u> background doping concentration, N_A+N_D)**

Today:

- **Diffusion <u>with recombination</u>**
- **The diffusion length (distance until they recombine)**

Assume holes (p) are minority carriers.

Consider simple volume element where we have both generation, recombination, and holes passing through due to a concentration gradient (dp/dx).



This is simple "bean counting" in the little volume.

Rate of bean population increase = (current IN – current OUT) – bean recombination

Or replace "bean" with "bubble" in little fish tank volume.

Note, this technique is very powerful (and often used) in *any* Finite Element (FE) computational or mathematical model.

So, let's count beans (bubbles):

- **Recombination rate = #excess bubbles (δp) / recombination time (τ)**
- **Current (#bubbles) IN – Current (#bubbles) OUT = J_{IN} – J_{OUT} / dx**

Note units (VERY important check):

Bubble current is bubbles/cm^2/second, but bubble rate of change (G&R) is bubbles/cm^3/second.

So must account for width (dx ~ cm) of volume slice.

$$\frac{\partial p}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J}{\partial x} - \frac{\partial p}{\tau}$$

Why does the first equality hold? (it's simple, boring math)

Why is there a (diffusion) current derivative divided by q?

Of course, for (e.g.) holes,

$$J_{DIFF} = \qquad\qquad\qquad \text{so,}$$

$$-\frac{1}{q}\frac{\partial J}{\partial x} =$$

So the diffusion equation (which is just a special case of the continuity equation above) becomes:

$$\frac{\partial \delta p}{\partial t} = D_P \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau}$$

This allows us to solve for the minority carrier concentration *in space and time* (here, holes).

Note, this is applicable only for minority carriers, whose net motion is dominated by diffusion.

What does this mean in *steady state*?

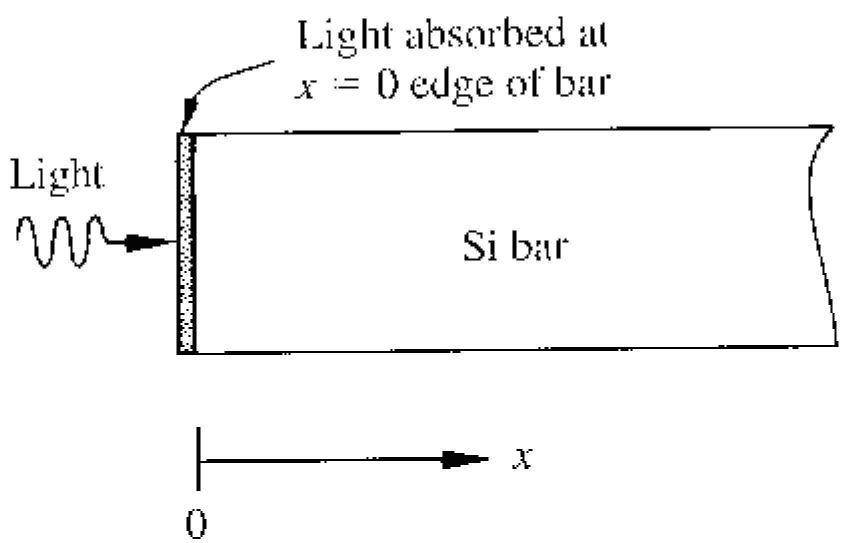
The diffusion equation in steady state:

$$\frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{D_P \tau} =$$

Interesting: this is what a lot of other diffusion problems look like in steady state. Other examples?

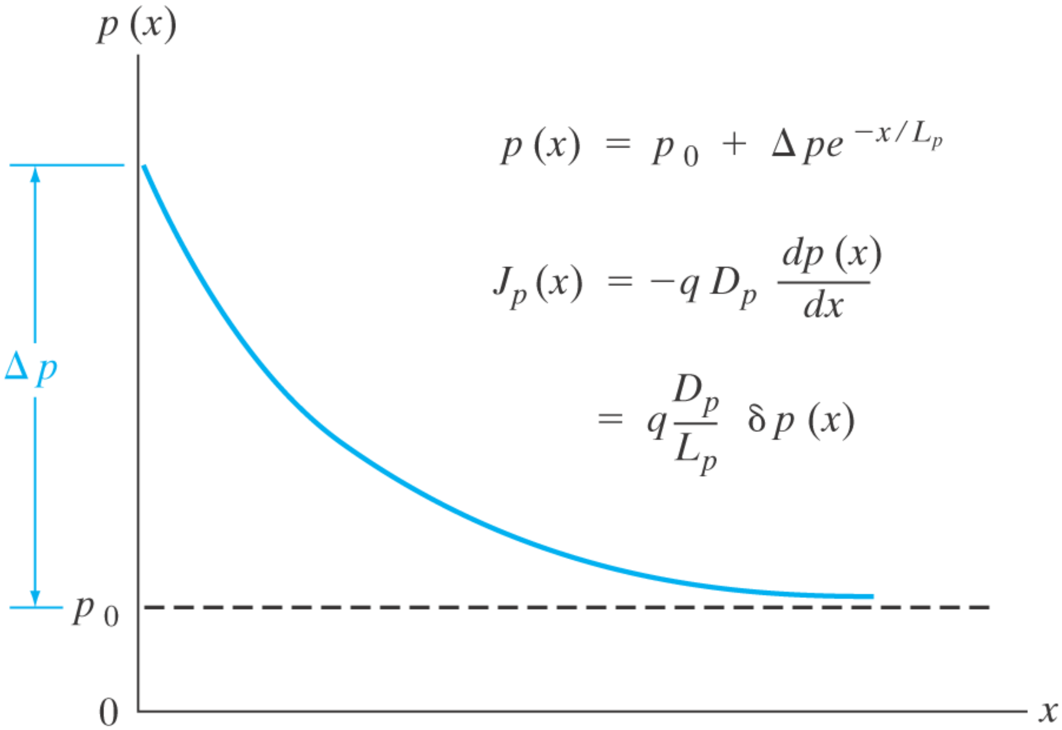
The *diffusion length* L_p = is a figure of merit.

Consider an example under steady-state illumination:



Solve diffusion equation: $\delta p(x) = \Delta p\, e^{-x/L_p}$

Plot:



Physically, the diffusion lengths (L_P and L_N) are the average distance that minority carriers can diffuse into a sea of majority carriers before being annihilated (recombining).

What devices is this useful in?! (peek ahead)

Example: A) Calculate the minority carrier diffusion length in silicon with $N_D = 10^{16}$ cm^{-3} and $\tau_p = 1$ μs. B) Assuming 10^{15} cm^{-3} excess holes photogenerated at the surface, what is the diffusion current at 1 μm depth?